Math 5A Q4 2.2-2.5 20 points

Key points from section:

Simplify $f'(x)$ *includes:*

No complex fractions, No negative exponents, Combine fractions

Label $f(x)$, $f'(x)$

Sometimes it is easier to simplify $f(x)$ *before differentiating*

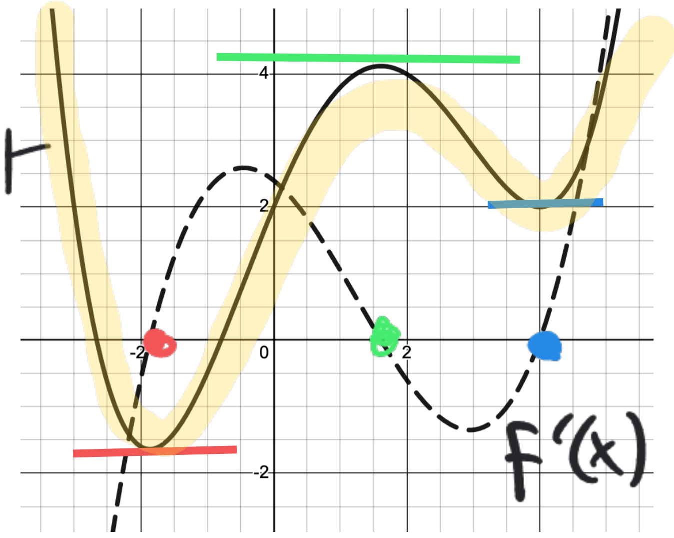
Notice, you can check your answers to derivatives using online apps, but that may not replace your work.

(1) The graphs below are of a function and its derivative. Clearly label which is $f(x)$ and which is $f'(x)$ (2 points)

$f'(x) = 0$

$\Rightarrow$ horizontal tangent on f

$\Rightarrow$ x-intercept on $f'(x)$



f(x)

2) Differentiate the following functions and simplify (4 points)

(a) $f(x) = \dfrac{3x^4 - 5x^3 + 7x}{x^2}$

* easiest of simplify first

$f(x) = 3x^2 - 5x + 7x^{-1}$

$f'(x) = 6x - 5 - 7x^{-2}$

$f'(x) = 6x - 5 - \frac{7}{x^2}$

$$\boxed{f'(x) = \frac{6x^3 - 5x^2 - 7}{x^2}}$$

(b) $g(t) = \sqrt{9 - t^2} = (9-t^2)^{1/2}$

chain rule

$g'(t) = \frac{1}{2}(9-t^2)^{-1/2} \frac{d}{dt}(9-t^2)$

$g'(t) = \frac{1}{2}(9-t^2)^{-1/2}(-2t)$

$$\boxed{g'(t) = \frac{-t}{\sqrt{9-t^2}}}$$

3) Differentiate the following functions and simplify (6 points)

c) $h(x)=\frac{\sqrt[3]{x}}{x-3}$

d) $f(x)=\tan(x)\cos(x^3)$

Quotient rule

$$h'(x)=\frac{(x-3)\frac{d}{dx}x^{1/3}-x^{1/3}\frac{d}{dx}(x-3)}{(x-3)^2}$$

$$h'(x)=\frac{(x-3)\frac{1}{3}x^{-2/3}-x^{1/3}}{(x-2)^2}$$

I factor out $\frac{1}{3}x^{-2/3}$

$$=\frac{\frac{1}{3}x^{-2/3}\left[(x-3)-3x\right]}{(x-2)^2}$$

$$\boxed{h'(x)=\frac{-3-2x}{3x^{2/3}(x-2)}}$$

product rule

$$f'(x)=\frac{d}{dx}\tan(x)\cos(x^3)+\tan x\frac{d}{dx}\cos(x^3)$$

chain

$$=\sec^2x\cos(x^3)+\tan x\left(-\sin x^3\frac{d}{dx}(x^3)\right)$$

$$=\sec^2x\cos(x^3)-\tan x\sin(x^3)(3x^2)$$

$$\boxed{f'(x)=\sec^2x\cos(x^3)-3x^2\tan x\sin x^3}$$

slope = 1/2

4). Find point(s) on the graph of $f(x)=\dfrac{x-1}{x+1}$ for which the tangent line is parallel to the line $x-2y=5$ then use a computer graph to illustrate your conclusion as thoroughly as possible. Attach screen shot.

(8 points)

Since we are not given the point of tangency, call it $P(a, f(a)) = P\left(a, \frac{a-1}{a+1}\right)$

The slope of the given line is $m = 1/2$

so we need $f'(a) = \frac{1}{2}$

<u>Find $f'(x)$</u> quotient rule

$$f'(x) = \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

So $f'(a) = \frac{1}{2} \Rightarrow \frac{2}{(a+1)^2} = \frac{1}{2}$

$$(a+1)^2 = 4$$

$$a+1 = \pm 2$$

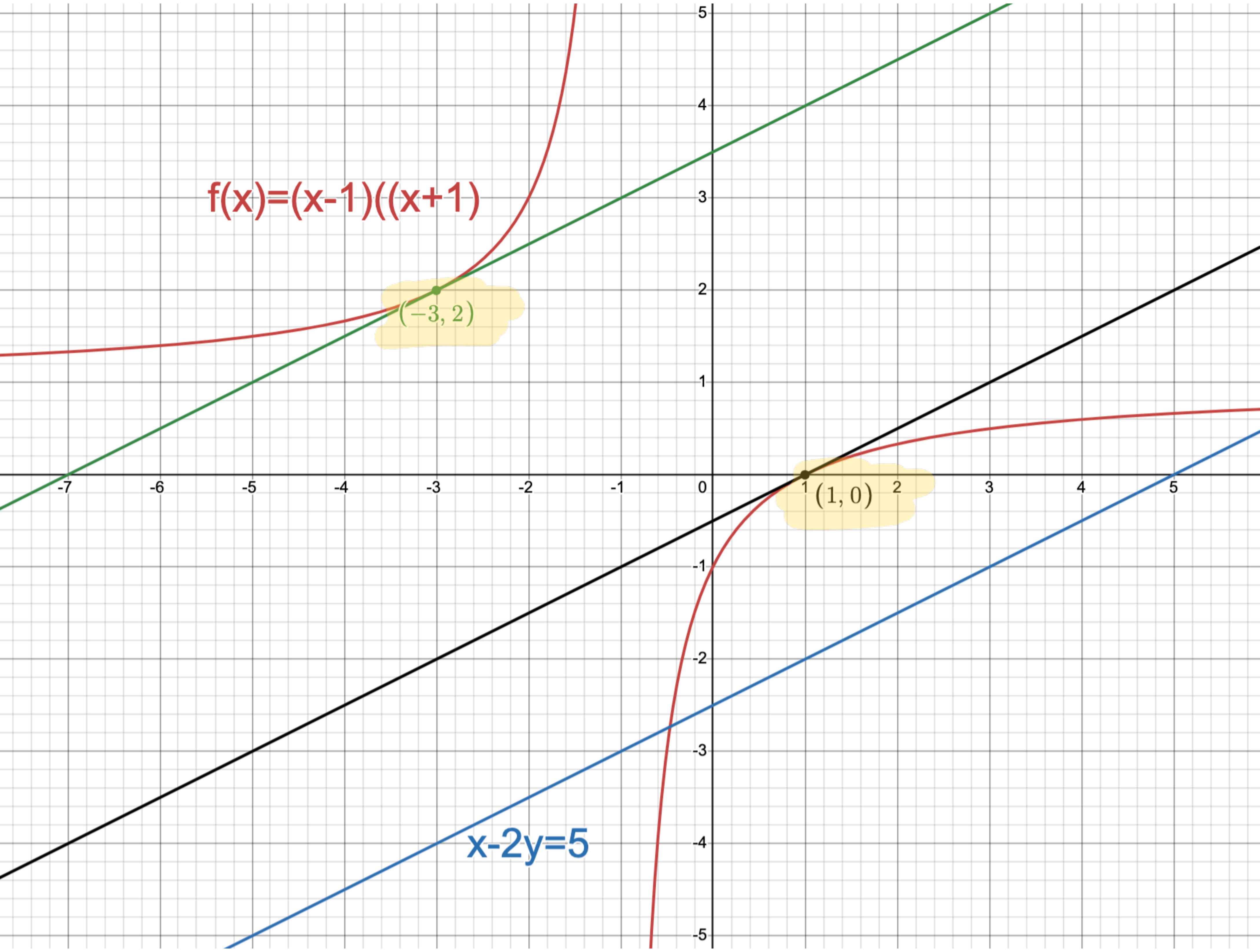
$$a = -1 \pm 2$$

$a = 1$, $a = -3$

$\Rightarrow$ points

$(1, f(1))$ $\quad$ $(-3, f(-3))$

$\boxed{(1, 0)}$ $\quad$ $\boxed{(-3, 2)}$



f(x)=(x-1)((x+1)
(−3, 2)
(1, 0)
x-2y=5